PAPER AAS 93-572

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AAS/AIAA Astrodynamics Specialist Conference

VICTORIA, B.C., CANADA AUGUST 16-19, 1993

AAS Publications Office, P.O. Box 28130, San Diego, CA 92198

ACCURACY ASSESSMENT OF TDRSS-BASED TOPEX/POSEIDON ORBIT DETERMINATION SOLUTIONS*

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Orbit determination results are obtained by the Goddard Space Flight Center (GSFC) Flight Dynamics Division (FDD) using the Goddard Trajectory Determination System (GTDS) and a real-time extended Kalman filter estimation system to process Tracking Data and Relay Satellite (TDRS) System (TDRSS) measurements in support of the Ocean Topography Experiment (TOPEX)/Poseidon spacecraft navigation and health and safety operations. GTDS is the operational orbit determination system used by the FDD, and the extended Kalman filter was implemented in an analysis prototype system, the Real-Time Orbit Determination System/Enhanced (RTOD/E). The Precision Orbit Determination (POD) team within the GSFC Space Geodesy Branch generates an independent set of high-accuracy trajectories to support the TOPEX/Poseidon scientific data. These latter solutions use the Geodynamics (GEODYN) orbit determination system with laser ranging and DORIS tracking data.

The TOPEX/Poseidon trajectories were estimated for the October 12 through November 21, 1992, the timeframe for which the latest preliminary POD results were available. Independent assessments were made of the consistencies of solutions produced by the batch and sequential methods. The batch least-squares solutions were assessed using overlap comparisons, while the sequential solutions were assessed with the estimated covariances and the first measurement residuals. The batch least-squares and forward-filtered RTOD/E orbit solutions were compared with the definitive POD orbit solutions. The solution differences were generally less than 7 meters for the batch least-squares and less than 18 meters for the sequential estimation solutions. The differences among the POD, GTDS, and RTOD/E solutions can be traced to differences in modeling and tracking data types, which are being analyzed in detail.

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* This work was supported by the National Aeronautics and Space Administration (NASA)/Goddard Space Flight Center (GSFC), Greenbelt, Maryland, under Contract NAS 5-31500.

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1.0 INTRODUCTION

This paper assesses the Ocean Topography Experiment (TOPEX)/Poseidon orbit determination accuracy of the Tracking and Data Relay Satellite (TDRS) System (TDRSS)-based orbit solutions using an operational batch least-squares system and a prototype sequential orbit determination system at the Goddard Space Flight Center (GSFC) Flight Dynamics Division (FDD). The TDRSS-based orbit solutions are compared with the preliminary high-precision orbit solutions obtained by the GSFC Space Geodesy branch using laser and Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS) tracking measurements.

The National Aeronautics and Space Administration (NASA) has completed a transition from tracking and communications support of low Earth-orbiting satellites with a ground-based station network, the Ground Spaceflight Tracking and Data Network (GSTDN), to the geosynchronous relay satellite network, the TDRSS. TDRSS consists of four operational geosynchronous spacecraft and the White Sands Ground Terminal (WSGT) at White Sands, New Mexico. The ground network provided only about 15-percent visibility coverage, while TDRSS can provide 85-percent to 100-percent coverage, depending on spacecraft altitude.

The Bilateration Ranging Transponder System (BRTS) provides range and Doppler measurements for determining each TDRS orbit. The ground-based BRTS transponders are tracked as if they were TDRSS-user spacecraft. Since the positions of the BRTS transponders are known, their ranging data can be used to precisely determine the trajectory of the TDRSs.

The accuracy requirements on the Space Geodesy Branch Geodynamics[1] (GEODYN) orbit determination solutions, used to analyze the sea surface height measurements obtained by the TOPEX/Poseidon radar altimeter, are extremely stringent. The definitive orbit determination requirements for the TOPEX/Poseidon mission science data include a maximum 13-centimeter radial position error. The accuracy of the precision orbit ephemerides (POEs) is being verified through the use of the TOPEX/Poseidon science data. Radar altimeter measurements over known overflight verification sites and the ocean surface are taken and then compared with coincident definitive TOPEX ephemerides generated using the ground-based laser and DORIS tracking. The GEODYN force modeling is then calibrated to minimize the differences between the definitive TOPEX ephemerides and the radar altimeter measurements.

Preliminary high-accuracy ephemerides, with an accuracy better than 13 centimeters in the radial direction, will be used to assess the accuracy of FDD-generated orbit determination solutions. The availability of the orbit determination solutions generated by the Space Geodesy Branch provides a unique opportunity to evaluate the accuracy of the orbit determination systems used by the FDD for operational and analysis navigation support.

This paper assesses the TDRSS-based orbit determination accuracy in the batch least-squares method that is used for operational orbit determination support in the GSFC Flight Dynamics Facility (FDF). The paper also assesses the accuracy of a sequential method implemented in a prototype system, used for analysis in the FDF. The batch weighted least-squares algorithm implemented in the Goddard Trajectory Determination System[2] (GTDS) estimates sets of orbital elements, force modeling parameters, and measurement-related parameters that minimize the summed squared differences between observed and calculated values of selected tracking data over a solution arc.

The sequential estimation algorithm implemented in a prototype system, the Real-Time Orbit Determination/Enhanced (RTOD/E),[3] simultaneously estimates the TDRSS user and relay spacecraft orbital elements and other parameters in the force and observation models at each measurement time.[4] RTOD/E performs forward filtering of tracking measurements using the extended Kalman filter with a process noise model to account for serially correlated, geopotentially induced errors, as well as Gauss-Markov processes for drag, solar radiation pressure, and measurement biases. The main features of RTOD/E can be found in an earlier paper.[5]

The estimated TOPEX/Poseidon ephemerides were obtained for the period October 12 through November 21, 1992. This timeframe was chosen because it was the latest for which the preliminary Precision Orbit Determination (POD) results were available. Independent assessments were made to examine the internal consistencies of results obtained by the batch and sequential methods.

This paper describes the orbit determination and evaluation procedures used in this study, summarizes POD solutions,[6] describes the results obtained by the batch least-squares and sequential estimation methods, provides the resulting consistency and cross comparisons, and presents the conclusions of this study.

2.0 ANALYSIS PROCEDURES

This section describes the analysis procedures used in this study and provides a description of the tracking measurements and orbit determination and modeling methods.

2.1 Tracking Measurements

The TOPEX/Poseidon spacecraft was launched aboard an Ariane 42P expendable launch vehicle in August 1992. In October 1992, maneuvers were completed that moved the spacecraft into its operational orbit, which is circular with an inclination of 66 degrees, an altitude of 1336 kilometers, a period of 112 minutes, and a 10-day ground track repeat period. The time period chosen for this study was from 23:30 hours coordinated universal time (UTC) on October 12, 1992, through 17:30 hours UTC on November 21, 1992, which corresponds to the third through sixth 10-day ground track repeat cycles, hereafter referred to as Cycles 3 through 6, respectively.

Tracking measurements from the TDRSS, used for TOPEX/Poseidon operational orbit navigation support by the FDF, were used to generate the GTDS and RTOD/E ephemerides. The GTDS orbit solutions were obtained using one-way and two-way Doppler data. For the first 4 days of Cycle 3, the RTOD/E orbit solutions were obtained using two-way range and Doppler data but no one-way data. For the remainder of the 40-day period, RTOD/E solutions were based on one-way and two-way Doppler data but no two-way range data. Throughout the 40-day period, RTOD/E processed BRTS range and Doppler data in addition to TDRSS data.

During Cycles 3 through 6, there were three TDRSs actively tracking user spacecraft; however, at any given time, only two TDRSs tracked TOPEX. The three TDRSs were TDRS-West (TDRS-5, 174 degrees west longitude), TDRS-East (TDRS-4, 41 degrees west longitude), and TDRS-Spare (TDRS-3, 62 degrees west longitude). TDRS-1 was not tracking user spacecraft.

The tracking consisted of an average of 10 passes of one-way Doppler observations and 11 passes of two-way Doppler observations per day, with the average pass lasting 40 minutes. A representative daily TDRSS tracking data distribution from Cycle 4 is shown in Figure 1. Passes labeled "2" consist of two-way Doppler observations, while passes labeled "1" consist of one-way Doppler observations. BRTS tracking coverage of each TDRS spacecraft typically consists of twelve to fifteen 5-minute passes per day.

The POD team used ground-based laser ranging and Doppler measurements from the DORIS system to generate the POEs. The laser tracking data network consists of approximately 50 ground stations located around the world. Fifteen of these stations are specifically designated to support TOPEX/Poseidon tracking. Most of the stations are located in the United States, Europe, and Australia. Table 1 shows the number of ground stations used and tracking passes obtained for Cycles 3 through 6.

A typical pass of laser ranging data lasts from 10 to 15 minutes. Table 2 shows the number of laser tracking data passes and observations for each day in Cycle 4, which was representative.

The DORIS tracking system, developed by the Centre Nationale d'Etudes Spatiales (CNES), consists of a global network of approximately 50 ground-based tracking beacons that provide one-way ground-to-spacecraft Doppler tracking measurements. For each cycle, tracking data were obtained from approximately 40 of these ground beacons, which generated a total of about 1300 tracking passes per cycle. Each pass is approximately 10 minutes in duration.

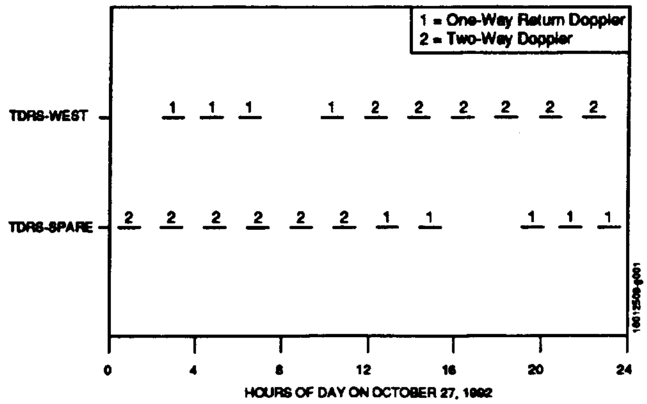

Figure 1 TDRSS Tracking Data for TOPEX

Table 1

NUMBERS OF GROUND STATIONS AND TRACKING PASSES FOR CYCLES 3 THROUGH 6

Cycle	Number of Laser Stations	Number of Tracking Passes
3	20	187
4	19	154
5	24	167
6	21	183

Table 2

NUMBERS OF LASER TRACKING DATA PASSES AND OBSERVATIONS FOR EACH DAY IN CYCLE 4

Day	Passes	Observations
10/22/92	3	55
10/23/92	20	501
10/24/92	15	247
10/25/92	25	487
10/26/92	14	233
10/27/92	15	324

Day	Passes	Observations
10/28/92	18	286
10/29/92	11	249
10/30/92	15	247
10/31/93	10	184
11/01/92	8	184

2.2 Orbit Determination Methods and Modeling

This section describes the orbit determination methods and the modeling used to generate the TOPEX/Poseidon solutions and ephemerides and provides the orbit determination methods and modeling for the POEs, GTDS batch least-squares solutions, and RTOD/E sequential estimation solutions.

2.2.1 Precision Orbit Ephemerides. The POEs are generated by the Space Geodesy Branch POD team using the GEODYN program. Each POE spans a 10-day period coincident with a project-defined beginning and end of a repeatable ground track. GEODYN, like GTDS, uses a batch least-squares estimation process to fit the tracking data and estimate a solution.

At the time of this analysis, the POD team was analyzing and improving the accuracy of the POEs and had not finalized the GEODYN force modeling. Therefore, the POEs used in this study are preliminary and do not represent the quality of the final POEs to be used to support the TOPEX/Poseidon science data. The quality of the preliminary POEs is discussed later in the paper.

The POEs used in this analysis cover the period from 23:50 hours UTC on October 12, 1992, through 17:30 hours on November 21, 1992. This time span covers four 10-day groundtrack repeat cycles corresponding to Cycles 3 through 6. Table 3 lists the epochs and time spans for the POEs.

Table 3

POE SOLUTION EPOCHS AND TIME SPANS

Cycle	Epoch [Date and Time UTC]	Time Span [Date and Time UTC]
3	10/12/92 23:50	10/12/92 23:50 – 10/22/92 23:29
4	10/22/92 19:33	10/22/92 19:33 – 11/01/92 21:30
5	11/01/92 17:32	11/01/92 17:31 – 11/11/92 21:30
6	11/11/92 15:30	11/11/92 15:29 – 11/21/92 17:29

The important force models and parameters used in the preliminary POEs are given in Table 4. The TOPEX/Poseidon dynamic solve-for parameters consist of the TOPEX spacecraft state vector, one once-per-revolution along-track acceleration per day, one once-per-revolution cross-track acceleration per day, and one constant along-track acceleration per day. These once-per-revolution along-track and cross-track accelerations were introduced to better model an anomalous spacecraft body-fixed acceleration discovered shortly after launch. Atmospheric drag and solar radiation forces are applied but are not solved for. The constant along-track acceleration was introduced as an adjustment for atmospheric drag.

Table 4

FORCE MODELING AND PARAMETERS USED IN THE PRELIMINARY POEs

Orbit Determination Parameter or Option	POE Values
Estimated parameters	Orbital state, along-track acceleration, cross-track acceleration
Integration type	11th-order fixed-step Cowell
Coordinate system of integration	True-of-reference
Integration step size	30.0 seconds
Tracking data	Ground-based laser ranging and DORIS data
Data rate	1 per 30 seconds
Differential correction convergence parameter	2 percent between iterations
Editing criterion	3.5 σ
Satellite area model	Box/wing model
Geopotential model	70 × 70 Joint Gravity Model-1 (JGM-1)
Atmospheric density model	Drag temperature model (DTM)
Coefficient of atmospheric drag	2.3
Coefficient of solar radiation pressure	1.0
Solar and lunar ephemerides	JPL Developmental Ephemeris-200 (DE-200)
Tropospheric refraction correction	Yes
Polar motion correction	Yes
Solid Earth tides	Yes
Ocean tides	Yes
Plate motion	Yes
Earth radiation pressure	Yes

2.2.2 Batch Least-Squares Estimation. The batch least-squares estimation algorithm used by GTDS for this analysis is the same as that used for operational navigation support of the TOPEX/Poseidon mission by the GSFC FDF. The procedure for operational support includes solving for the spacecraft state, onboard ultrastable oscillator (USO) bias and drift parameters, and an along-track thrust estimation parameter using two-way and one-way Doppler measurements. Range measurements are excluded from the solutions to avoid limitations in solving for uncorrected biases, which have been found to reduce the orbit solution quality. The modeling and state solve-for parameters used for this analysis have been enhanced to provide more accurate results and to take advantage of modeling and techniques not currently in operational use. Specifically, the state space was expanded to include estimation of the coefficient of solar radiation pressure and nominally 30-hour along-track thrust parameters that were intended to compensate for the anomalous acceleration on the spacecraft. Also, the data span was extended to cover one entire 10-day ground track repeat period, as opposed to the 7-day, 10-hour solution spans used operationally. The modeling and options used are presented in Table 5, and Table 6 lists the epochs and data spans.

Table 5

PARAMETERS AND OPTIONS USED IN THE GTDS SOLUTIONS

Orbit Determination Parameter or Option	GTDS Values*	
	TOPEX	TDRS-East, TDRS-West, TDRS-Spare
Estimated parameters	Orbital state, thrust coefficients (τ, one/30 hours), coefficient of solar radiation pressure (C_R), USO bias and drift	Orbital state, coefficient of solar radiation pressure (C_R), BRTS range bias
Integration type	Cowell 12th order	Cowell 12th order
Coordinate system of integration	Mean-of-J2000.0	Mean of J2000.0
Integration step size (seconds)	60 seconds	600 seconds
Tracking measurements	TDRSS two-way Doppler TDRSS one-way return Doppler	BRTS two-way range
Data span	10 days	See text
Data rate	1 per minute	1 per 10 seconds
DC convergence parameter	0.00005	0.005
Editing criterion	3σ Central angle of 79.48 degrees	3σ
Measurement weight sigmas	0.25 Hz two-way, 0.13 Hz one-way	10 meters
Satellite area model	Variable mean area model	Constant, 40 meters^2
Satellite mass	2417.2 kilograms	See Table 8
Geopotential model	50×50 GEM-T3	20×20 GEM-T3
Atmospheric density model	Jacchia-Roberts	N/A
Solar and lunar ephemerides	DE-200	DE-200
Coefficient of drag (C_D)	2.3 applied	N/A
Ionospheric refraction correction Ground-to-spacecraft Spacecraft-to-spacecraft	 Yes No (central angle edit instead)	 Yes N/A
User-spacecraft antenna offset correction	Constant radial	No
Tropospheric refraction correction	Yes	Yes
Polar motion correction	Yes	Yes
Solid Earth tides	Yes	Yes
Ocean tides	No	No
Plate motion	No	No
Earth radiation pressure	No	No

*GEM = Goddard Earth Model; Hz = hertz; N/A = not applicable

Table 6

GTDS SOLUTION EPOCHS AND DATA SPANS

Cycle	Epoch	Data Span
3	10/13/92	10/13/92 – 10/23/92
3.5	10/18/92	10/18/92 – 10/28/92
4	10/23/92	10/23/92 – 11/02/92
4.5	10/28/92	10/28/92 – 11/07/92
5	11/02/92	11/02/92 – 11/12/92
5.5	11/07/92	11/07/92 – 11/17/92
6	11/12/92	11/12/92 – 11/22/92

The TDRS orbits used to process the one-way return and two-way TDRSS Doppler data used in the TOPEX batch estimation process were obtained from solutions for TDRS-East, TDRS-West, and TDRS-Spare that were determined separately using only BRTS tracking. Covariance analysis corresponding to previous orbit determination results indicated that the TDRS orbit solutions were a primary contributor to the error in the TOPEX orbit estimation. As indicated by the covariance analysis, the use of BRTS two-way range-only tracking, instead of the operational range and Doppler tracking, resulted in improved TDRS orbit determination. A range bias solve-for solution was found to reduce the effects of WSGT ranging calibration errors on the TDRS orbit solutions. Additionally, the TDRS solution arcs were selected to avoid all maneuvers and angular momentum wheel unloads. The improvements made to the TDRS orbit determination resulted in reduced TOPEX orbit determination differences when compared with the POEs.

Analysis of the operational TOPEX/Poseidon orbit solutions has indicated the presence of an unmodeled spacecraft body-fixed force with a day-to-day variability. Analysis performed by the Jet Propulsion Laboratory (JPL) has indicated that the unmodeled force is dependent on the angle between the orbit plane and the Sun.[7] Consequently, in addition to an applied drag force, a series of thrust scale factors (referenced to a 1-micronewton continuous along-track thrust) was estimated. Distribution of the thrust scale factors was nominally one per 30 hours, with exceptions made for changes in the spacecraft solar array configuration and attitude flight mode. The choice of one correction factor per 30 hours was driven by software limitations. Table 7 lists the TDRS maneuvers and spacecraft events occurring during the solution arcs.

Table 7

TDRS AND TOPEX EVENTS DURING THE GTDS SOLUTIONS

Date and Time of Event	Event
10/15/92 19:05 UTC	TDRS-3 north/south maneuver
10/16/92 07:10 UTC	TDRS-3 north/south maneuver
10/19/92 13:06 UTC	TOPEX steering mode change to fixed yaw
10/20/92 02:15 UTC	TOPEX steering mode change from fixed to sinusoidal yaw
10/20/92 20:35 UTC	TDRS-3 east/west maneuver
10/26/92 13:58 UTC	TDRS-5 north/south maneuver
10/27/92 02:25 UTC	TDRS-5 north/south maneuver
11/02/92 13:00 UTC	TDRS-4 replaces TDRS-3 for TOPEX support
11/04/92 00:48 UTC	TDRS-4 east/west maneuver
11/13/92 08:14 UTC	TOPEX steering mode change to fixed yaw
11/19/92 02:14 UTC	TOPEX yaw flip maneuver
11/19/92 11:40 UTC	TDRS-5 east/west maneuver

The GTDS solutions were evaluated on the basis of a series of overlapping 10-day solutions, one every 5 days, resulting in a 5-day overlap. The epochs were placed at the start of the data arcs, and the definitive ephemeris overlap position comparisons were used to judge the solution-to-solution consistency. The tracking data residual statistics and comparison of corresponding solution solve-for parameters were also used to evaluate the GTDS solutions.

2.2.3 Sequential Estimation. In this work, RTOD/E serves as a research tool for assessing sequentially estimated orbit solutions generated within a realistic FDF environment. RTOD/E execution has been in progress since TOPEX was launched in August 1992. During some portions of the period leading up to Cycle 3, RTOD/E was in a real-time or near-real-time operating mode. At various points, execution was suspended to accommodate maneuvers and adjust tuning parameters. In addition, complete reinitialization of RTOD/E was necessary on several occasions.

At the start of Cycle 3, the filter had been running since September 3, 1992, 00:00:00 UTC, over 1 month before the start of the cycle. It had been initialized for TOPEX, TDRS-3, and TDRS-5 at that epoch. The filter was reinitialized for the same three satellites on October 17, 1992, 00:00:00 UTC (day 4 of Cycle 3) to accommodate one-way Doppler tracking measurements and adjustments to some of the tunable parameters. The filter was reinitialized for TOPEX, TDRS-4, and TDRS-5 on November 2, 1992, 00:00:00 UTC, which nearly coincided with the start of Cycle 5. Two generic initial orbit RIC [radial, in-track (along-track), and cross-track] covariance matrices, one for TOPEX and one for the TDRSs, were used for each initialization.

Tables 8 and 9 provide detailed information on of the models and options used. The RTOD/E solution state included orbital elements for TOPEX and each of two TDRSs. Other estimated quantities included a coefficient of atmospheric drag for TOPEX, a coefficient of solar radiation pressure for each of the three satellites, Doppler measurement biases, and the USO bias. The USO oscillator bias is modeled as a random-walk process with a linear drift term. The full RTOD/E state error covariance matrix had a dimension of 27 by 27 when not processing BRTS measurements. During BRTS passes, the measurement biases for BRTS range and Doppler measurements are added to the state space.

A comparison between the RTOD/E and POD ephemerides, resolved in orbit-plane principal directions, provided the primary means of gauging the sequential orbit determination accuracy of this analysis. The comparisons were performed in the J2000.0 true-of-date (TOD) coordinate frame. Other indicators of RTOD/E solution quality were provided by the diagonal elements of the state error covariance matrix,[8] the integrity of the drag coefficient estimates, and the relationship of the first predicted residual to the residual standard deviation for each tracking pass.

3.0 RESULTS AND DISCUSSION

This section presents the TOPEX/Poseidon accuracy assessment analysis results, an assessment of the consistency of the TOPEX/Poseidon ephemerides, and the ephemeris comparison results.

3.1 Accuracy Assessment of the POEs

To support the science objectives of the TOPEX/Poseidon mission, the POD team is required to produce POEs that are accurate to 13 centimeters in the radial component. Comparisons of the preliminary POEs with actual TOPEX/Poseidon radar altimeter data show agreement to within 12 centimeters in the radial component. These comparisons, in conjunction with a battery of other verification tests, provide strong evidence that the POEs are sufficiently accurate to meet the 13-centimeter requirement. In addition, the tests also indicate that the along-track and cross-track components are approximately as accurate as the radial component.[6]

One aspect of the POE verification involves performing overlap comparisons to assess solution consistency between the POEs and specially generated overlap solutions. Five special overlap solutions were generated and compared for the preliminary POEs for Cycles 3 through 6. Each overlap solution spans the last 5 days of the previous cycle and the first 5 days of the following cycle. The results show an average root-mean-square (RMS) overlap radial position difference of 3 centimeters, which is less than one-fourth the magnitude of the 13-centimeter accuracy requirement.

Table 8

PARAMETERS AND OPTIONS FOR SIMULTANEOUS TOPEX AND TDRS SOLUTIONS

Orbit Determination Parameter or Option	RTOD/E Values			
	TOPEX		TDRS-East (E)/West (W)/Spare (S)	
	Baseline*	Changes**	Baseline*	Changes**
Estimated parameters	Orbital state, coefficient of drag, TDRSS range and Doppler tracking, measurement biases	Baseline + coefficient of solar radiation pressure, USO bias	Orbital state, coefficient of solar radiation pressure, BRTS range and Doppler tracking, measurement biases	–
Integration type	VOP	–	VOP	–
Coordinate system of integration	Mean of 1950.0	–	Mean of 1950.0	–
Integration step size	60.0 seconds	–	600.0 seconds	–
Tracking data	TDRSS two-way range and Doppler	TDRSS two-way and one-way Doppler	BRTS range and Doppler	–
Data rate	1 per minute	1 per 30 seconds	1 per minute	1 per 30 seconds
Editing criterion	3σ	–	3σ	–
Gravity error autocorrelation values	R: 2.828 minutes I: 0.001 minutes C: 5.611 minutes Errors of omission and commission	–	N/A	–
Measurement sigmas: Range Doppler	 0.40 meters 0.02 hertz	 N/A 0.010 hertz	 0.40 meters 0.02 hertz	 0.50 meters 0.010 hertz
Gauss-Markov parameters: Drag half-life Drag sigma C_R half-life C_R sigma Range bias half-life Range bias sigma Doppler bias half-life Doppler bias sigma	 840.0 minutes 1.000 N/A N/A 60.0 minutes 6.0 meters 8 minutes 0.034 hertz	 – 0.400 1440.0 minutes 0.200 N/A N/A – –	 N/A N/A 11520.0 minutes 0.0003 60.0 minutes 7.0 meters 60.0 minutes 0.030 hertz	 – – - 0.200 – – – –
GM standard deviation	0.005 km^3/sec^2	–	0.005 km^3/sec^2	–
Satellite area	32 $meters^2$	–	40 $meters^2$	–
Satellite mass	2430.842 kg	2417.200 kg	1824.98 kg (S) 1982.02 kg (W)	1853.6 kg (E) 1973.1 kg (W)
Satellite identification number	9205201	–	8809102 (S) 9105402 (W)	8902102 (E) 9105402 (W)
Nominal USO frequency	N/A	19.056 megahertz	N/A	N/A
USO fractional noise standard deviation	N/A	5×10^{-10}	N/A	–
USO deweighting standard deviation	N/A	10^{-13}	N/A	–
USO deweighting time constant	N/A	10.0 seconds	N/A	–

NOTES: * Baseline—Prior to October 17, 1992.

** Changes—From October 17, 1992, 00:00:00 UTC through the end of Cycle 6: (a) USO data incorporated; (b) Filter reinitialized on November 2, 1992, 00:00:00 UTC, when TDRS-East replaced TDRS-Spare.

– = No change from Baseline

Abbreviations: VOP = Variation of Parameters; km = kilometers; sec = seconds; kg = kilograms

Table 9

FORCE AND MEASUREMENT MODEL SPECIFICATIONS

Model or Option	RTOD/E Values	
	TOPEX	TDRS-East/West
Geopotential model	GEM-T3 (50 × 50)	GEM-T3 (8 × 8) (truncated)
Atmospheric density model	CIRA 72*	N/A
Solar and lunar ephemerides	Analytic	Analytic
Coefficient of drag	Estimated with a priori value of 1.25	N/A
Coefficient of reflectivity	Estimated with a priori value of 1.4 (after 10/17/92)	Estimated with a priori value of 1.4
Ionospheric refraction correction	No	No
Tropospheric refraction correction	Yes	Yes
Antenna mount correction	No	No
Polar motion correction	Yes	Yes
Earth tides	No	No

*CIRA = Committee on Space Research (COSPAR) International Reference Atmosphere

3.2 Summary of the Batch Least-Squares Estimation Results

Figure 2 summarizes the RMS and maximum position differences during the overlap periods. The mean and sample standard deviation of this distribution, in the form of mean ± standard deviation, is 2.1 ± 0.7 meters, with a maximum RMS difference of 3.1 meters. The overlap maximum position differences are nearly constant at 3 meters, with the exception of the Cycle 4–Cycle 4.5 and Cycle 5.5–Cycle 6 overlap time periods, where the comparisons are nearly 6 meters. For Cycle 4–Cycle 4.5, the maximum along-track and cross-track differences are nearly equal at approximately 4.5 meters. This overlap is the fault of the Cycle 4.5 solution, which had a change of TDRSs in the middle of the solution and had poorer measurement residual statistics. The large cross-track comparison value supports this, since the TOPEX orbit-plane errors are sensitive to the TDRS orbit errors, which will be different for each TDRS due to the specific BRTS tracking geometry used. The change in TDRSs would then introduce some inconsistencies with the adjoining solutions.

The other large overlap maximum position comparison, during the Cycle 5.5–Cycle 6 overlap, is dominated by the along-track position difference. The along-track difference is the result of a significant difference in the solved-for thrust coefficients during the time period of elevated along-track thrust resulting from the attitude mode change that occurred around November 13. Figure 3 gives the solved-for thrust coefficients, of the form $(1 + \tau)$ times the nominal 1-micronewton applied thrust.

Overall, the average maximum position differences were 3.6 ± 1.3 meters. Generally, the maximum position differences are influenced by both along-track and cross-track position differences. One of the high-overlap cases is the result of a change in TDRSs used for TOPEX tracking, indicating that further improvements are still possible in the TDRS orbit determination. The other case is the result of differences in the solved-for thrust coefficients, indicating that further refinements can be made in the thrust modeling.

As a result of the altimetric goals of the TOPEX/Poseidon mission, the radial accuracy of the precision ephemerides used for the science processing is 13 centimeters. The maximum and RMS overlap radial differences are given in Figure 4. The RMS values varied from 20 centimeters to just over 40 centimeters, with the maximum differences ranging from 35 to 95 centimeters. The average RMS radial position difference is 0.32 ± 0.07 meters, which is approximately three times the requirement for the precision ephemerides.

The comparisons of the overlap velocity differences are presented in Figure 5. The distribution of the maximum and RMS differences is virtually identical to the total position overlaps, as was expected. The average overlap total velocity difference RMS is 1.5 ± 0.5 millimeters/second, and the average maximum is 3.4 ± 1.0 millimeters/second. Once again, the average RMS value is representative of the consistency over the entire overlap period.

Solution measurement residuals for the one-way and two-way Doppler tracking data used are presented in Figure 6. For all of the solutions, the two-way Doppler residual statistics are generally consistent, with the mean of the residuals averaging -1.6 ± 1.6 megahertz and the standard deviation of the residuals averaging 27.4 ± 5.3 megahertz. The mean residual is much smaller than the standard deviation, indicating that no significant biases exist in the measurements. Because the USO bias and drift were estimated, the mean one-way residuals were expected to be insignificant, and the resulting mean residual was virtually zero. The standard deviation of the one-way residuals was approximately 60 percent of that of the two-way residuals, averaging 18.2 ± 2.3 megahertz. Because the one-way data travel only one of the paths of the two-way data, the one-way data noise is expected to be greater than 50 percent of the two-way noise if the processes on the two paths are not fully correlated.

In summary, the enhanced GTDS solutions show an average RMS consistency of 2.1 meters in position over the definitive data arcs. At the ends of the data arcs, maximum variations reached 6.0 meters. The differences are attributable in part to differences in the solve-for thrust estimation factors near the end of the arcs and to modeling inconsistencies, such as TDRS position errors, that produce cross-track position errors.

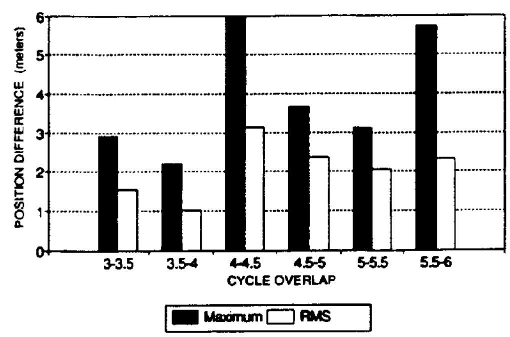

Figure 2 Overlap Comparisons

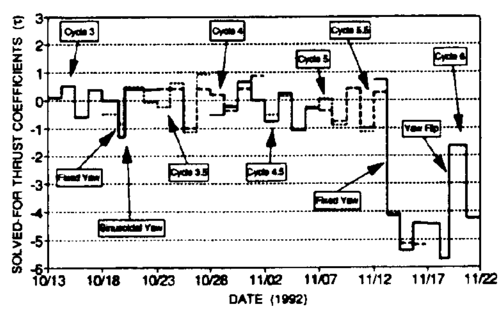

Figure 3 Estimated Thrust Coefficients

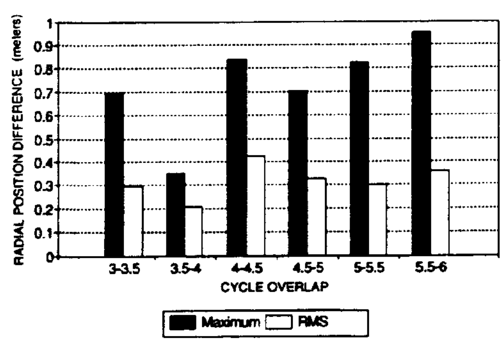

Figure 4 Radial Overlap Comparisons

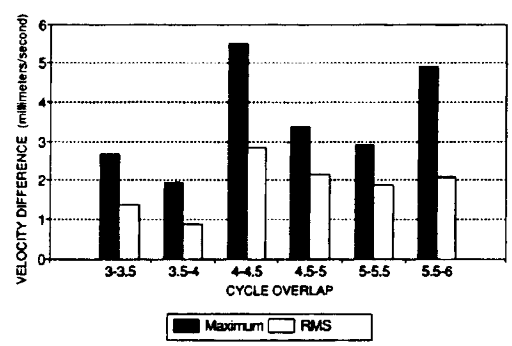

Figure 5 Velocity Overlap Comparisons

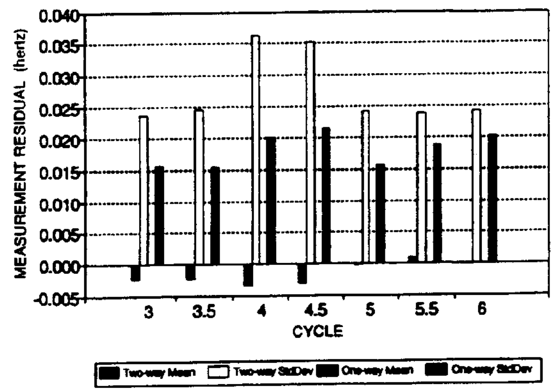

Figure 6 Doppler Residual Statistics

3.3 Summary of Sequential Estimation Results

Several indicators were available to assess the quality of the RTOD/E solutions independent of other orbit determination systems. Among such performance criteria are the diagonal components of the state error covariance matrix, more specifically, the square root of these values (standard deviation). Figures 7 and 8 show the time-evolution of the 1σ root-sum-square (RSS) position error for each of three satellites during the Cycle 4 period and during the Cycle 5/6 period, respectively, as computed by RTOD/E. Table 10 presents the average 3σ position errors for TOPEX for Cycles 3 through 6. Portions of Cycles 3 and 5 in the vicinity of a reinitialization or a TOPEX maneuver are not reflected in the averages.

The relative constancy in the TOPEX root variance shown in the figures indicates that RTOD/E solutions had largely settled from earlier maneuver perturbations and initial condition errors. The two spikes apparent in the TDRS-5 profile in Figure 7 correspond to a pair of burns used for an orbit-plane maintenance maneuver. The smaller spike apparent in Figure 8 corresponds to a single burn used for longitudinal stationkeeping. These sudden changes in the orbital state root variance were direct consequences of the application of an assumed velocity change (delta-V) uncertainty to the covariance matrix. As reflected in the impulsive character of the variations, the recovery time was minimal. Overall, a gradual trend toward reduced levels in standard deviations is seen for each relay satellite.

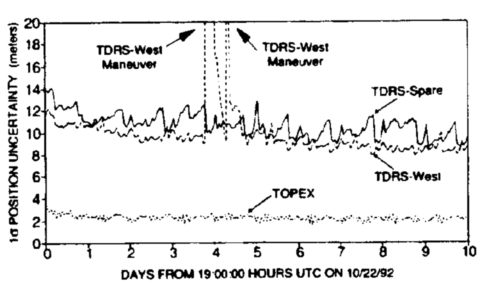

Figure 7 1σ Position Uncertainty [Cycle 4]

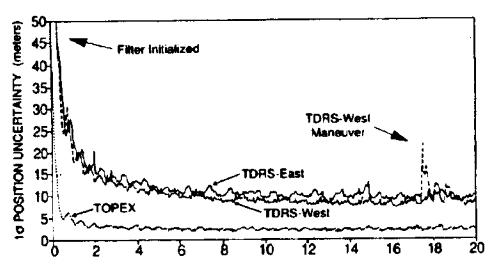

Figure 8 1σ Position Uncertainty [Cycles 5 and 6]

Table 10

AVERAGE 3σ POSITION ERROR FOR TOPEX

Component	3σ Position Error (meters)			
	Cycle 3*	Cycle 4	Cycle 5**	Cycle 6
RSS	10.6	6.6	7.7	6.4
Radial	2.4	1.3	1.3	1.3
Along-Track	9.0	5.4	6.2	5.3
Cross-Track	4.7	3.6	4.3	3.2

* Excluding day 4

** Excluding the first 2 days

Additional evidence of solution consistency is provided by the size of the predicted residual for the first tracking measurement in a pass. The residuals for the first TDRSS one-way and two-way Doppler measurements for each tracking pass on a typical day (November 6) are provided in Figure 9 . Each residual shown in the figure is within the 3σ bound in the residual space.

Trends in the estimates for the coefficients of the solar radiation pressure (TDRS and TOPEX) and the coefficient of atmospheric drag (TOPEX) can be seen in Figures 10 and 11, respectively, which cover Cycles 5 and 6. Similar behavior was observed in the coefficient estimates for Cycle 4 and the portion of Cycle 3 after the October 17 reinitialization. When properly tuned, the estimated values of the drag and solar radiation coefficients should accommodate mismodeling of the atmospheric density and uncompensated variations in the solar radiation force model, respectively. It should be noted that the relative uncertainties in the modeling of atmospheric densities are greater for spacecraft at higher altitudes than for spacecraft at lower altitudes. In addition to atmospheric modeling and solar flux level uncertainties, changes in the spacecraft attitude can be expected to induce variation in the coefficient estimates (RTOD/E uses a constant-area cross-section for both drag and solar radiation pressure computations). Given these factors, the observed variation in the coefficient estimates was judged to be reasonable.

Finally, the USO bias estimate was used as a basis for performance assessment. RTOD/E estimates the shift in the S-band TDRSS transmit frequency induced by the USO bias as a fraction of a nominal value of the transmit frequency. The nominal S-band TDRSS transmit frequency used for RTOD/E execution was 2106.40625 megahertz. Figure 12 shows the USO clock frequency scale factor estimate from RTOD/E for a

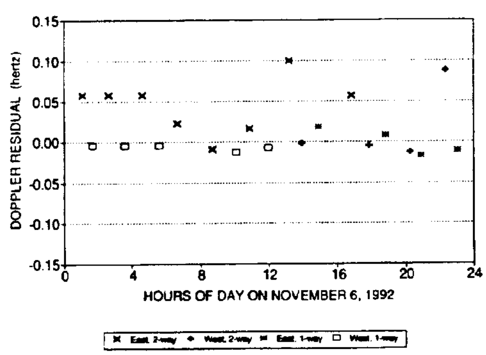

Figure 9 **Residual Values for the First Measurements of Each TOPEX Tracking Pass [November 6, 1992]**

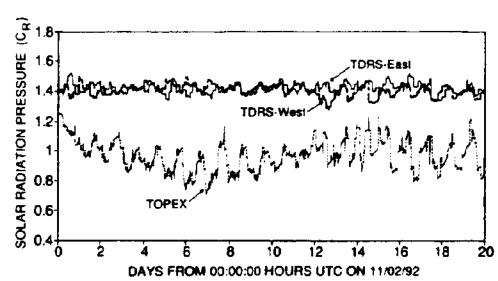

Figure 10 **Coefficients of Solar Radiation Pressure [Cycles 5 and 6]**

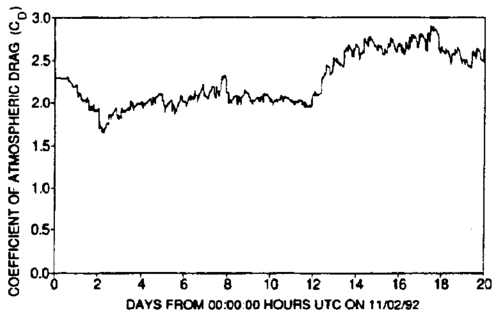

Figure 11 Coefficients of Atmosphere Drag for TOPEX [Cycles 5 and 6]

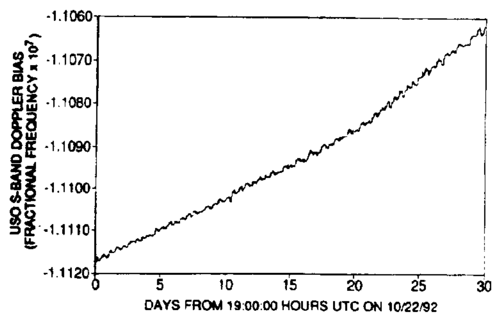

Figure 12 USO Bias Estimate From RTOD/E Solutions

30-day period covering Cycles 4 through 6. The trend is generally consistent with corresponding estimates generated independently by the batch least-squares orbit determination process. The change in slope that is apparent near day 18 is not well understood but is believed to be a consequence of the presence of unmodeled frequency drift.

3.4 Results of POE and GTDS Solution Comparisons

Four 10-day GTDS ephemerides, corresponding to Cycles 3 through 6, were compared with the respective POEs. The time spans of the GTDS definitive ephemerides are given in Table 6.

The ephemerides were compared at 10-minute intervals in orbit plane coordinates on their common definitive spans. The RSS position differences between the GTDS ephemerides and the POEs for Cycles 3 through 6 are shown in Figure 13. The average RSS position difference is 2.9 meters, with a maximum difference of 6.7 meters.

Figure 14 shows the representative differences in the radial, cross-track, and along-track directions on November 6, 1992. The maximum radial difference is 0.9 meter, while the maximum cross-track difference is 2.4 meters. The maximum along-track difference, which is the largest of the three components, is about 3.8 meters. The differences in the along-track component have an average value of 1.9 meters, while the average differences in the radial and cross-track components are nearly zero.

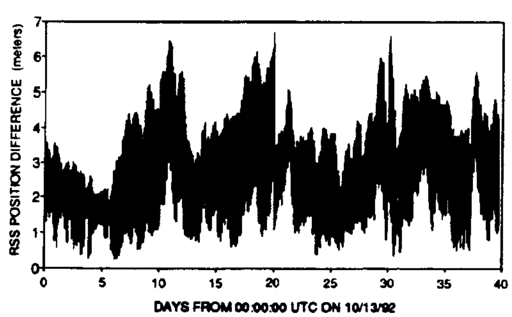

Figure 13 Position Differences Between POEs and GTDS Ephemerides [Cycles 3 Through 6]

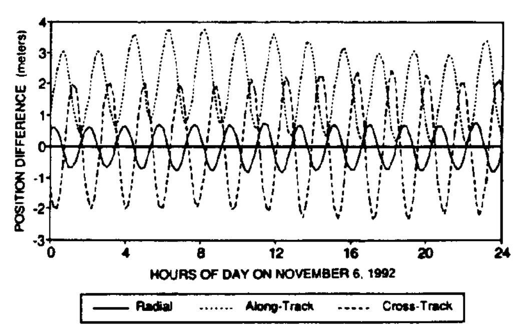

Figure 14 Position Differences by Component Between POE and GTDS Ephemerides [November 6, 1992]

Prior to the improvements to the TDRS solutions, cross-track differences between the POE–GTDS comparisons had dominated the position difference. Further analysis of that cross-track component showed that the smallest differences occurred at the orbital nodes, while the largest differences occurred at the maximum latitudes, indicating a disparity in inclination. Using the more accurate TDRS solutions contributes to a more accurate determination of the orientation of the TOPEX orbit plane. For Cycle 4, the maximum cross-track difference was reduced from 9.8 meters to 5.0 meters. Similar reductions in the cross-track differences were observed for Cycles 3, 5, and 6.

Some of the difference in the along-track component is likely due to differences in the modeling of along-track accelerations. The POEs estimate a daily once-per-revolution along-track acceleration and a daily constant along-track acceleration to accurately model the effects of the anomalous spacecraft forces. This represents a total of 20 solve-for parameters to characterize the along-track accelerations. The GTDS solutions, however, solve for only eight thrust scale factors to characterize the along-track forces.

3.5 Comparison Between POEs and Sequential Ephemerides

The ephemeris comparison results are illustrated in Figures 15 and 16. Figure 15 shows the RSS position difference between the POEs and the RTOD/E ephemerides for the 40-day period covered by Cycles 3 through 6. Figure 16 shows radial, cross-track, and along-track components of the position difference between the POEs and the RTOD/E ephemerides during a representative day Cycle 5 (November 6, 1992).

The position difference exhibits distinct characteristics for each of the four cycles shown in Figure 15. For Cycle 3, a maximum RSS position difference of 50 meters before the October 17 reinitialization and 15 meters after reinitialization is observed. At the beginning of Cycle 3, the effects of a TOPEX maneuver on the solution are clearly reflected in the comparison. A dramatic improvement in postinitialization solution quality is evident. To some degree, the improvement arose from refinements in the tunable parameters; however, expanded tracking coverage provided by the addition of one-way Doppler data was also a factor.

For Cycle 4, the position difference grows to approximately 19 meters near the middle of the 10-day cycle and reduces to approximately 10 meters near the beginning and end of the cycle. In addition, a slight 24-hour modulation of approximately 2 meters is visible. Furthermore, the total difference is dominated by the cross-track component, and a bias of approximately –2 meters was observed in the along-track component of the position difference.

After the November 2 reinitialization (Cycles 5 and 6), the overall agreement between the POEs and the RTOD/E ephemerides is improved. The improvement can perhaps be attributed to a more favorable tracking geometry after the TDRS reassignment. Further analysis is in progress. For Cycle 5, a maximum RSS position difference of 14 meters is observed after an initial settling period. The ephemeris difference is

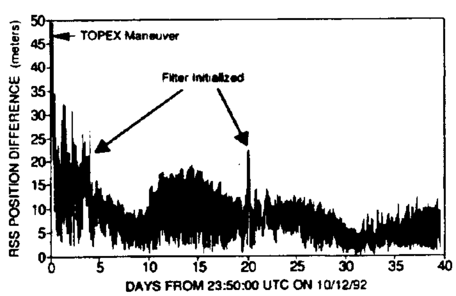

Figure 15 Position Difference Between POEs and RTOD/E Ephemerides [Cycles 3 Through 6]

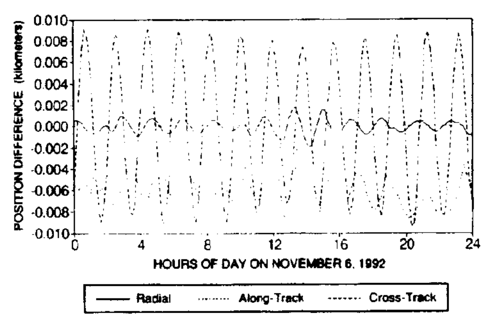

Figure 16 Position Difference Between POEs and RTOD/E Ephemerides [November 6, 1992]

influenced equally by cross-track and along-track components, as Figure 16 indicates. For Cycle 6, a maximum RSS position difference of 12 meters is observed. The difference is generally dominated by the along-track and cross-track components. Table 11 summarizes the ephemeris comparison results for the four cycles.

Table 11

SUMMARY OF EPHEMERIS COMPARISON RESULTS

Component	POSITION DIFFERENCES* (meters)							
	Cycle 3**		Cycle 4		Cycle 5**		Cycle 6	
	Max	Avg	Max	Avg	Max	Avg	Max	Avg
RSS	30.7	10.0	19.0	9.9	14.0	7.5	12.3	5.2
Radial	7.2	0.2	2.0	0.1	2.1	0.1	3.2	0.1
Along-Track	26.1	4.6	9.1	2.8	12.9	4.7	10.0	1.7
Cross-Track	17.7	0.0	18.6	0.0	9.8	0.0	10.0	0.0

* Max = maximum; Avg = average

** Excluding the first 2 days

In an attempt to identify the sources of cross-track discrepancy for Cycle 4, the comparison was repeated in an Earth-fixed coordinate frame for various portions of the 40-day period. For Cycle 4, the RSS position difference envelope was reduced by approximately 5 meters and exhibited no 24-hour modulation. Somewhat less improvement was observed for portions of other cycles. The benefit of using the Earth-centered, Earth-fixed (ECEF) coordinate frame was found to be greatest for intervals where the cross-track component dominated. This condition suggests that a significant portion of the ephemeris difference arose from discrepancies in Earth orientation modeling. Thus, the effects on the ephemeris comparison of Greenwich hour angle (GHA) and polar motion angle discrepancies were analyzed.

The cross-track discrepancy for Cycle 4 could not be appreciably reduced with a single rotation about the z-axis through an angle indicative of the mean GHA discrepancy. In addition, no appreciable change in the comparison result was found when the polar motion coefficients used to transform the RTOD/E orbital elements were matched to those specified in the POE header file. The precision in the coefficients that could be accommodated by RTOD/E (three significant figures), however, was inadequate for the test. The nature of discrepancies in the z-axis orientation were next examined by studying the trace made by the projection in the orbit plane of the orbit-normal vector difference arising between the POE and the RTOD/E ephemeris. The trace showed that an appreciable cross-track ephemeris difference could arise both for predominantly nodal and for predominantly polar orientations of the orbit normal differential. Thus, the z-axis orientation discrepancy could not be removed with a single rotation.

Additional insight into the origin of the remaining ephemeris difference is expected to grow from an examination of the simultaneous TDRS solutions provided by RTOD/E.

A comparison of the RTOD/E 3σ position uncertainties (see Table 10) with the maximum ephemeris differences (Table 11) provides an indication of how well tuned the filter is. The 3σ values indicate a lower error level than suggested by the comparison results by a factor of approximately 2 for Cycles 5 and 6 and by a factor of approximately 3 for Cycle 4. If allowance is made for systematic errors such as the coordinate system discrepancy discussed above, and because errors in the POEs are relatively negligible, then the uncertainty estimates provided by RTOD/E are slightly optimistic. Work is currently in progress to utilize the POEs to improve the RTOD/E tunable parameters for TOPEX.

3.6 Comparison Between the GTDS Batch Least-Squares and Sequential Ephemerides

The ephemeris comparison results are illustrated in Figures 17 and 18. Figure 17 shows the RSS position difference between a representative GTDS definitive ephemeris (solution epoch on November 2, 1992) and the RTOD/E ephemeris over Cycle 5. Figure 18 shows the radial, cross-track, and along-track components of the position difference over 1 particular day (November 6, 1992). After the initial settling of the RTOD/E solution, the RSS position difference reaches a maximum of approximately 17 meters. The RSS along-track, cross-track, and radial components average 8.3, –6.5, –0.03, and 0.9 meters, respectively. It should be noted that the coordinate frame inconsistency that exists between RTOD/E and GEODYN also exists between RTOD/E and GTDS. As an indication of this fact, the average GHA discrepancy between GTDS and GEODYN was approximately 1.4×10^{-8} radians, an order of magnitude smaller than the GEODYN–RTOD/E GHA discrepancy.

3.7 Remarks on Supporting Analysis

Batch least-squares covariance analysis was performed to analyze the GTDS solutions. The covariance analysis was performed corresponding to the GTDS solution with an epoch on November 2, 1992. The modeling for the covariance analysis was made as close as possible to the GTDS modeling. The 3σ RSS position uncertainty was found to vary between 7 and 15 meters. By components, the maximum 3σ position uncertainties were 3 meters, 5 meters, and 14 meters in the radial, cross-track, and along-track directions, respectively. The cross-track differences between the GTDS solutions and the POEs are less than the uncertainties obtained by covariance analysis. At the maximum 3σ RSS position uncertainty of 14.9 meters, the major contributors to the errors are the uncertainty in the ionospheric refraction correction at WSGT (11.6 meters) and the geopotential (6.0 meters).

The validity of the secular trends of the GTDS dynamic modeling was verified by performing GTDS solutions for arc lengths of 1 day through 10 days, with increasing arc lengths by a day each for Cycle 5. The characteristics of the comparison of the 10 solutions with the POEs did not change from the short (1-day) arc length to the long (10-day) arc length. Corresponding covariance analysis solutions with the same tracking schedules as the 10 GTDS solutions supported the GTDS solutions.

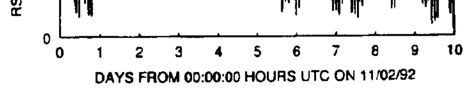

Figure 17 RSS Position Difference Between GTDS and RTOD/E Ephemerides [Cycle 5]

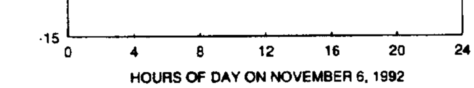

Figure 18 Component Position Difference Between GTDS and RTOD/E Ephemerides [November 6, 1992]

Several areas in the batch least-squares modeling and orbit determination processing could be improved to yield better results. First, the TDRS orbit determination can be further improved by better treatment of the BRTS ranging errors. Second, the area modeling of TOPEX itself should be improved. At present, only mean areas are used for the solar radiation and drag force computations. Also, the antenna offset model could be improved to incorporate the effects of the sinusoidal yaw steering mode. Finally, better treatment of the unmodeled body-fixed force should help improve the accuracy of the batch least-squares solutions.

It is important to note that TDRSS tracking does not have a requirement to yield orbit solutions with accuracy comparable to laser-tracked orbit solutions. However, a major objective of this work is to assess the TDRSS achievable orbit determination accuracy.

4.0 CONCLUSIONS

This study analyzed the TDRSS user orbit determination accuracy using a batch least-squares method and a sequential estimation method. Independent assessments were performed of the orbit determination consistency within each method, and the estimated orbits obtained by the two methods were compared.

In the batch least-squares analysis, the orbit determination consistency of GTDS solutions for TOPEX/Poseidon was found to be about 2 meters in the RMS overlap comparisons and about 5 meters in the maximum position differences in overlap comparisons. The differences between the definitive batch least-squares ephemerides and the POEs were no larger than 7 meters. The largest component in the differences was in the along-track direction, with the cross-track components being nearly as significant. The reduction of the cross-track differences, as compared with an earlier analysis,[9] was the direct result of the use of only range tracking in the TDRS orbit determination. This demonstrates that the treatment of the relay orbit determination has an impact on high-accuracy orbit determination in the TDRSS environment.

The sequential orbit solutions were within 20 meters of the POEs for a 36-day period (Cycles 4, 5, 6, and most of Cycle 3). The 3σ position uncertainties, which averaged 7 to 11 meters RSS, indicated a somewhat lower error level than suggested by the comparison results, even when allowance is made for the portion of the ephemeris difference attributable to coordinate transformation disparity. Analysis indicates that this coordinate transformation disparity, which was as much as 5 meters for Cycle 4, is not removable through a single coordinate frame rotation. Additional work involving refinements to 18 tunable parameters, smoothing techniques, and TDRS solution consistency is in progress. As a measure of consistency, the first residual of each pass was within the 3σ bound in the residual space.

The differences between the definitive batch least-squares ephemerides and the POEs were no larger than 7 meters. The differences between the forward-filtered sequentially estimated ephemerides and the POEs were no larger than 19 meters. Further analysis is in progress to understand the magnitudes of the differences. The differences among the POEs, GTDS, and RTOD/E solutions can be traced to differences in modeling and tracking data types, which are being analyzed in detail. As more precise POEs become available, further comparisons and analysis will be performed.

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